Data-driven Causal Effect Identification across Domains

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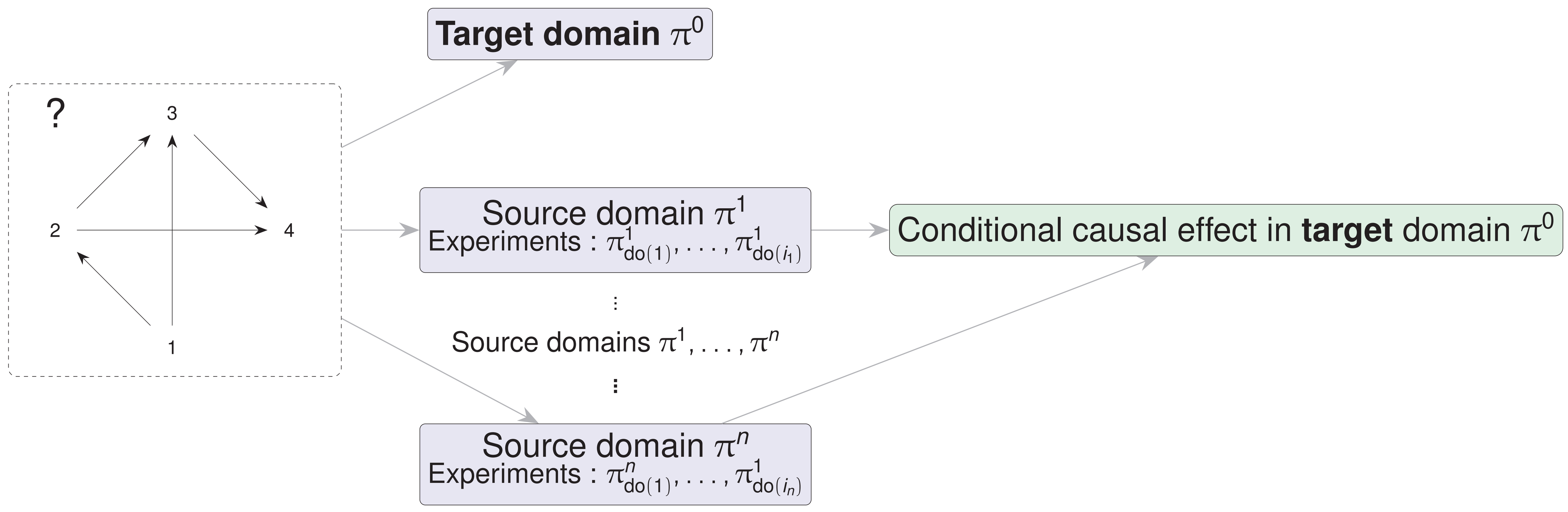
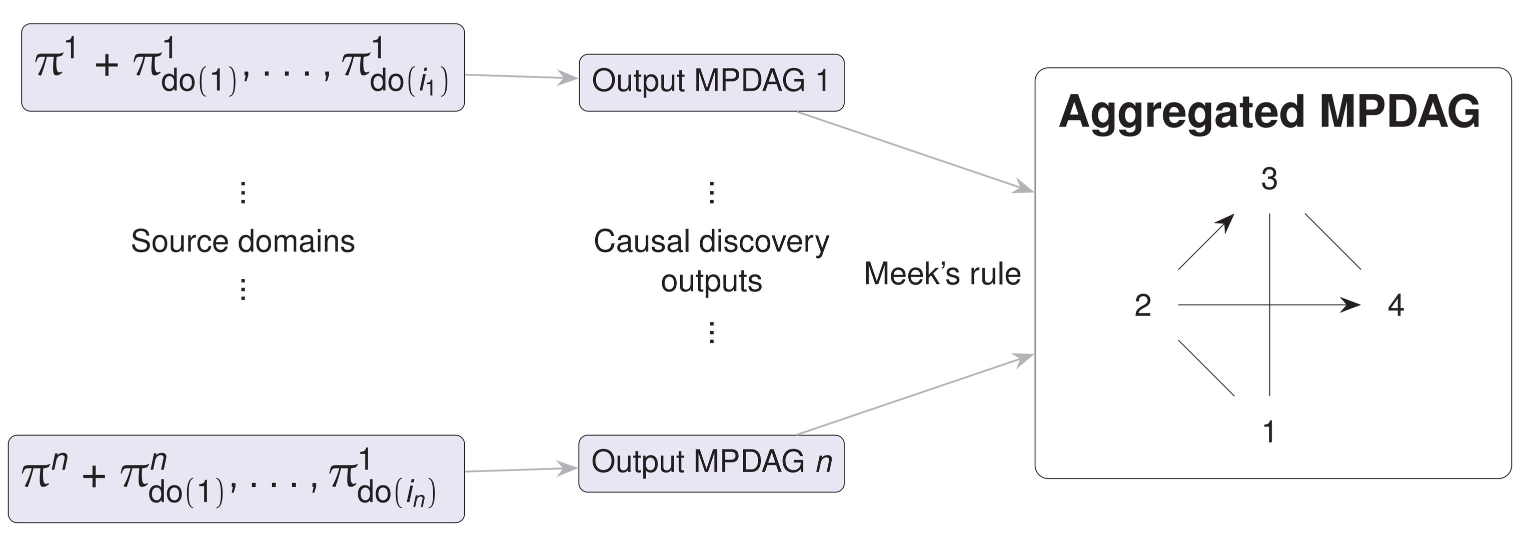
1. Motivation



Setup: Unknown causal DAG generates target domain π^0 and source domains $\pi^1, \dots, \pi^n$ via **structural causal models** with *discrepancies* between domains.

Objective: Using **observational + experimental** distributions from source domains $\pi^1, \dots, \pi^n$, identify conditional causal effects, e.g. CATE, in the target domain π^0.

2. Partially reconstruct the graph



Aggregate causal discovery outputs from observational + interventional distributions across source domains using **Meek's orientation rule** [Meek, 1995].

Partially recover the unknown causal DAG, represented as an MPDAG.

3. Encode domain discrepancies

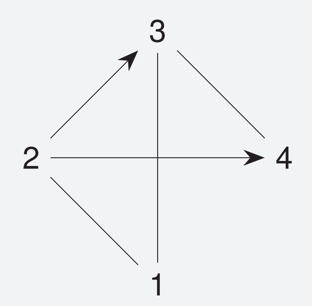
Target domain π^0	Discrepancy Δ_i	Source domain π^i
$X_k := f^0_k(X_{\text{pa}(k)}, \epsilon^0_k)$	$f^0_k \neq f^i_k$ or $\epsilon^0_k \neq_d \epsilon^i_k$ for all $k \in \Delta_i$	$X_k := f^i_k(X_{\text{pa}(k)}, \epsilon^i_k)$

Encode using a **selection diagram** $\mathcal{G}$ constructed from the **aggregated MPDAG**.

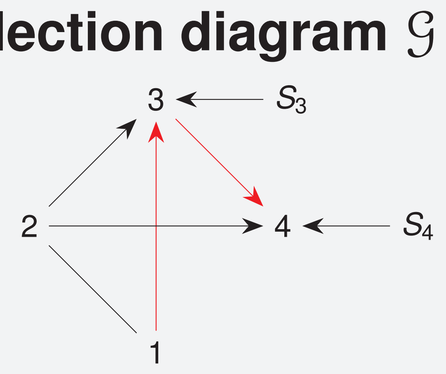
1. For all $k \in \Delta_i$ for some source domain π^i, **add** $S_k \to k$.
2. For such $k \in \Delta_i$, **orient** the edges $k - \ell$ as
   - $\ell \to k$, if $\pi^i(\ell)$ is the same for all domains π^i, or
   - $k \to \ell$, otherwise.
3. Apply **Meek's orientation rule** [Meek, 1995].

Example: for $\Delta_1 = \{3\}, \Delta_2 = \{4\}$

Aggregated MPDAG



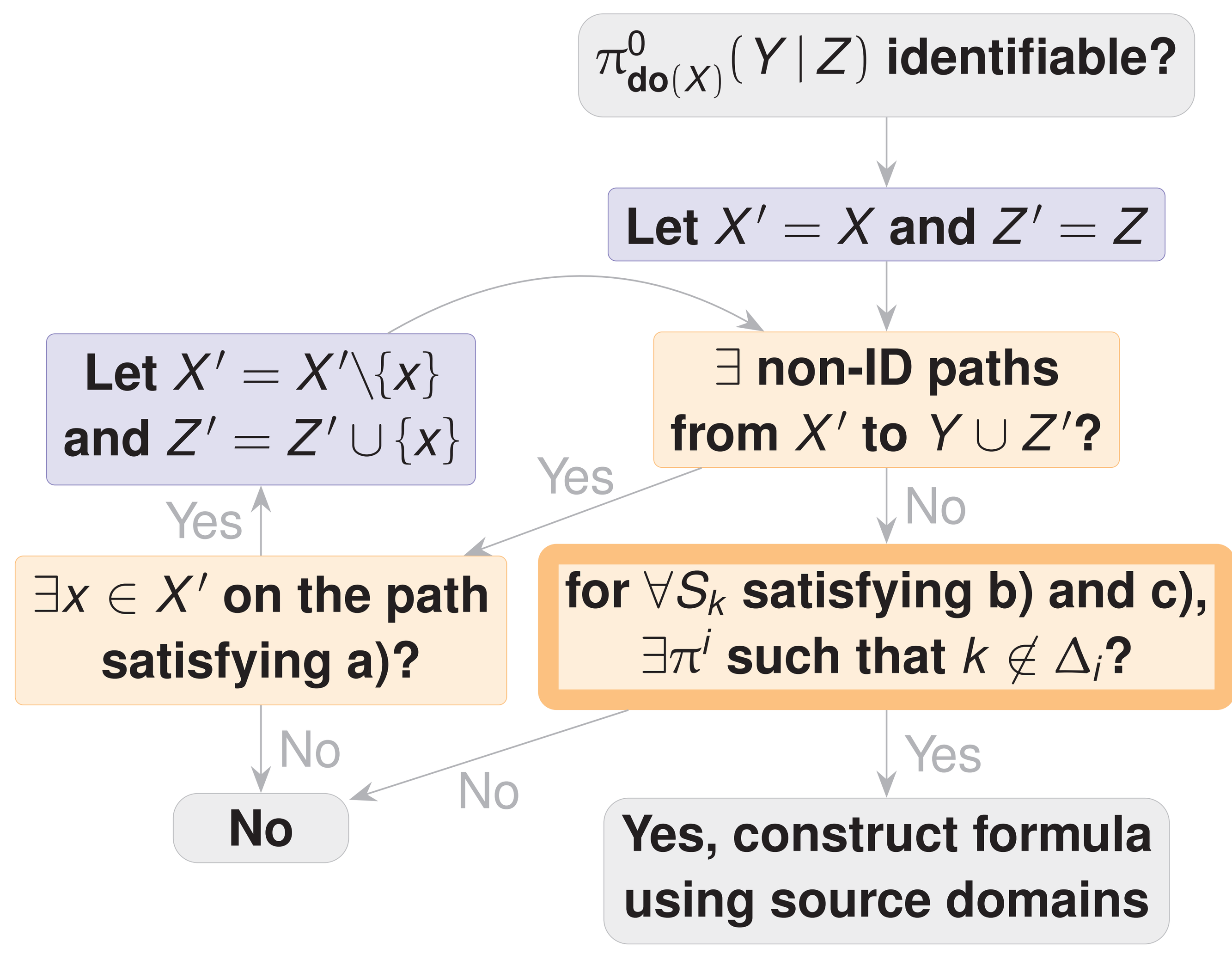
Selection diagram $\mathcal{G}$



References
1. Meek, C. Causal inference and causal explanation with background knowledge, 1995.
2. Laplante, S. and Perkovic, E. Identifying conditional causal effects from MPDAGs, 2025.

4. Identify from the selection diagram $\mathcal{G}$

Modifying Laplante and Perkovic 2025:



Intuition:

1. **Iteratively modify** the **treatment** set X and **conditioning** set Z into an identifiable form using the **while loop**.
2. **Check** for the required factors in the **source domains** to construct the formula for $\pi^0_{\text{do}(X)}(Y \mid Z)$.

$\mathcal{G}_{\overline{X}\underline{Z}}$: The graph $\mathcal{G}$ after removing edges $\to x \in X$ and $\leftarrow z \in Z$.

a) Y is d-separated from x given $Z' \cup X' \setminus \{x\}$ in $\mathcal{G}_{\overline{X' \setminus \{x\}}\underline{X}}$.
b) there is a directed path from S_k to $Y \cup Z$ in $\mathcal{G}_{\overline{X}}$.
c) S_k is **not** d-separated from Y given $X \cup Z$ in $\mathcal{G}_{\overline{X}}$.

Sound and **complete** algorithm!

TL; DR

Method to **identify conditional causal effects in the target domain from source domains** where the **underlying generating causal graph is unknown** and has to be inferred from data.

The method:

1. **Aggregates causal discovery outputs** across source domains, partially reconstructs the unknown graph as an **aggregated MPDAG**.
2. **Encodes** the discrepancy between target and source domains on a **selection diagram** constructed from the aggregated MPDAG.
3. **Determines** if the conditional causal effect is identifiable using the selection diagram, if so, **returns a formula** in terms of source domains.